Continuity and Differentiability
Part - 2

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $|\sin x|$ is continuous for all $x \in R$.

Reason (R): $\sin x$ and $|x|$ are continuous in R.

Ans. Option (A) is correct.

Explanation: $\sin x$ and $|x|$ are continuous in R. hence R is true.

Consider the functions $f(x) = \sin x$ and $g(x) = |x|$ both of which are continuous in R.

$$gof(x) = g(f(x)) = g(\sin x) = |\sin x|.$$

Since $f(x)$ and $g(x)$ are continuous in R, $gof(x)$ is also continuous in R.

Hence A is true.

R is the correct explanation of A.

Q. 2. Assertion (A): $f(x) = \tan^2 x$ is continuous at $x = \frac{\pi}{2}$.

Reason (R): $g(x) = x^2$ is continuous at $x = \frac{\pi}{2}$.

Ans. Option (D) is correct.

Explanation: $g(x) = x^2$ is a polynomial function. It is continuous for all $x \in R$.

Hence R is true.

$f(x) = \tan^2 x$ is not defined when $x = \frac{\pi}{2}$.

Therefore $f\left(\frac{\pi}{2}\right)$ does not exist and hence $f(x)$ is not continuous at $x = \frac{\pi}{2}$.

A is false.

Q. 3. Consider the function $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$

which is continuous at $x = 0$.

Assertion (A): The value of k is -3.

Reason (R): $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

Ans. Option (A) is correct.

Explanation:

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}.$$

This is the definition for modulus function and hence true.

Hence R is true.

Since f is continuous at $x = 0$,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

Here $f(0) = 3$,

$$\text{LHL} = \lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} \frac{kx}{|x|} = \lim_{x \to 0^-} \frac{kx}{-x} = -k$$

$\therefore$ $-k = 3$ or $k = -3$.

Hence A is true.

R is the correct explanation of A.

Q. 4. Consider the function

$$f(x) = \begin{cases} \frac{x^2 + 3x - 10}{x - 2}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$$

which is continuous at $x = 2$.

Assertion (A): The value of k is 0.

Reason (R): $f(x)$ is continuous at $x = a$, if

$$\lim_{x \to a} f(x) = f(a).$$

Ans. Option (D) is correct.

Explanation:

$f(x)$ is continuous at $x = a$, if $\lim_{x \to a} f(x) = f(a)$.

$\therefore$ R is true.

$$\lim_{x \to 2} f(x) = f(2) = k$$

$$\lim_{x \to 2} \frac{(x+5)(x-2)}{x-2} = k$$

$\therefore$ $k = 7$

Hence A is false.

Q. 5. Assertion (A): $|\sin x|$ is continuous at $x = 0$.

Reason (R): $|\sin x|$ is differentiable at $x = 0$.

Ans. Option (C) is correct.

Explanation: Since $\sin x$ and $|x|$ are continuous functions in R, $|\sin x|$ is continuous at $x = 0$.
Hence A is true.

$$|\sin x| = \begin{cases} -\sin x, & \text{if } x < 0 \\ \sin x, & \text{if } x \geq 0 \end{cases}$$

$$f(0) = |\sin 0| = 0$$

$$\text{LHD} = f'(0^-) = \lim_{x \to 0} \frac{-\sin x - 0}{x} = -1$$

$$\text{RHD} = f'(0^+) = \lim_{x \to 0} \frac{\sin x - 0}{x} = 1$$

At $x = 0$, LHD $\neq$ RHD.
So $f(x)$ is not differentiable at $x = 0$.
Hence R is false.

Q. 6. Assertion (A): $f(x) = [x]$ is not differentiable at $x = 2$.
Reason (R): $f(x) = [x]$ is not continuous at $x = 2$.

Ans. Option (A) is correct.

Explanation: $f(x) = [x]$ is not continuous when x is an integer.
So $f(x)$ is not continuous at $x = 2$. Hence R is true.
A differentiable function is always continuous.
Since $f(x) = [x]$ is not continuous at $x = 2$, it is also not differentiable at $x = 2$.
Hence A is true.
R is the correct explanation of A.

Q. 7. Assertion (A): A continuous function is always differentiable.
Reason (R): A differentiable function is always continuous.

Ans. Option (D) is correct.

Explanation: The function $f(x)$ is differentiable at $x = a$, if it is continuous at $x = a$ and

$$\text{LHD} = \text{RHD at } x = a.$$

A differentiable function is always continuous.
Hence R is true.
A continuous function need not be always differentiable.
For example, $|x|$ is continuous at $x = 0$, but not differentiable at $x = 0$.
Hence A is false.

Q. 8. Assertion (A): If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, then $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$

Reason (R): $\sin^{-1}(6x\sqrt{1-9x^2}) = 3\sin^{-1}(2x)$

Ans. Option (C) is correct.

Explanation:

put $3x = \sin\theta$ or $\theta = \sin^{-1} 3x$

$$y = \sin^{-1}(6x\sqrt{1-9x^2}) = \sin^{-1}(\sin 2\theta)$$
$$= 2\theta$$
$$= 2\sin^{-1} 3x$$

$$\therefore \quad \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$$

A is true. R is false.